Analytical Task Assignment

Belt Life Analysis

Jonathan Walgren

March 1 st 2019

18F02-Kinetic Sculpture

One of the newest additions to our Kinetic Sculpture is the use of timing belts instead of a gear-train to ensure the sculpture will operate for 20-30 minutes. This is a very large change when compared to our original proposed design. The reasons for this change include how much more reliable timing belts are, the need to perfectly align the gear-train so it won't "lock" itself up, and once we had to abandon our attempt to cast all the gears from molten aluminum we needed to find a more cost effective solution. As a result we needed to do an analysis to determine the expected lifetime of each individual belt. To find the hours of operation I used a series of machine design formulas were used.

Equation (1) uses the number of belt passes (N_p), pitch length (L_p), peripheral speed of the belt (V) to calculate the lifetime of the belt in hours (t). Equation (2) uses belt durability parameters (K) and (b) as well as the tension in the belt (T) to calculate the number of belt passes (N_p). Equation (3) determines the peripheral speed of the belt (V) using the small pulley diameter (d) as well as the RPM of the small pulley (n).

$$t = \frac{(Np*Lp)}{720V} \tag{1}$$

$$Np = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1} \tag{2}$$

$$V = \frac{\pi dn}{12} \tag{3}$$

Equation (4) allows you to calculate the tension in the belt from the largest belt tension (F_1) and the maximum tensile stress ($(F_b)_1$). We can use equation (5) to find the largest belt tension (F_1) using the centrifugal tension (F_c), power transmitted per belt (ΔF), and the angle of wrap ($f\phi$). In order to find the maximum tensile stress ($(F_b)_1$), we use equation (6) along with belt parameter (K_b).

$$T_1 = F_1 + (F_b)_1 \tag{4}$$

$$F_1 = F_c + \frac{\Delta F * e(f\phi)}{e(f\phi) - 1} \tag{5}$$

$$(F_b)_1 = \frac{K_b}{d} \tag{6}$$

The centrifugal tension (F_c) is found from equation (7) using the belt parameter (K_c) and the peripheral speed of the belt (V). Equation (8) uses the design horsepower (H_d), number of belts (N_b), the small pulley diameter (d) and the RPM of the small pulley (n) to find the power transmitted per belt (ΔF). Equation (9) finds the angle of wrap ($f\phi$) using the small pulley diameter (d), large pulley diameter (D), and the center to center distance (C). Finally equation (10) allows us the find the center to center distance using the pitch length (L_p), the small pulley diameter (d) and the large pulley diameter (D).

$$F_c = K_c\left(\frac{V}{1000}\right)^2 \tag{7}$$

$$\Delta F = \frac{630 * H_d / N_b}{n * (d/2)} \tag{8}$$

$$f\phi = \pi - 2 * sin^{-1}\frac{(D-d)}{2*C} \tag{9}$$

$$C = .25\{[L_p - \tfrac{\pi}{2}(D+d)]\} + \sqrt{[L_p - \tfrac{\pi}{2}(D+d)]^2 - 2*(D-d)^2\}} \tag{10}$$

Variables that are not calculated include the pitch length (L_p), durability parameters (K) and (b), small pulley diameter (d), large pulley diameter (D), RPM of small diameter pulley (n), belt parameters (K_b) and (K_c), design horsepower (H_d) and the number of belts (N_b). The pitch length is sized for each individual belt and was determined from the SolidWorks design that was created. The durability parameters are determined to be 674 and 11.089 from a table that specifies belt durability parameters for given sizes. The small pulley diameters and the large pulley diameters were determined to be 1 inch and 2.4 inches from the SolidWorks drawing in order to give the correct final drive ratio. The RPM of the pulleys were calculated using the belt ratios between each belt and a final RPM of 75 of the worm gear. The belt parameters are determined to be 220 and .561 from a table that specifies belt parameters for given sizes. The design horsepower is normally used from the motor powering the system however because the sculpture will spin slow this was set at .01 . Using the equations (1) thru (10) and the predetermined variables, each belt was individually calculated and the results are below in Table (1).

Table 1: Belt Life in Hours

Belt Life In Hours	
Belt 1	10729.40
Belt 2	14118.94
Belt 3	4557.09
Belt 4	2165.90
Belt 5	1195.24
Belt 6	406.70

From the information in Table 1, and knowing the operation time of the sculpture for each cycle will be 20 minutes, we can see that the sculpture will be able to complete approximately 1200 cycles before it will need a belt change. To combat this extra belts will be purchased so any belts that break can be easily replaced The Pulley alignment can be seen in Figure 1 which is a screenshot from SolidWorks.

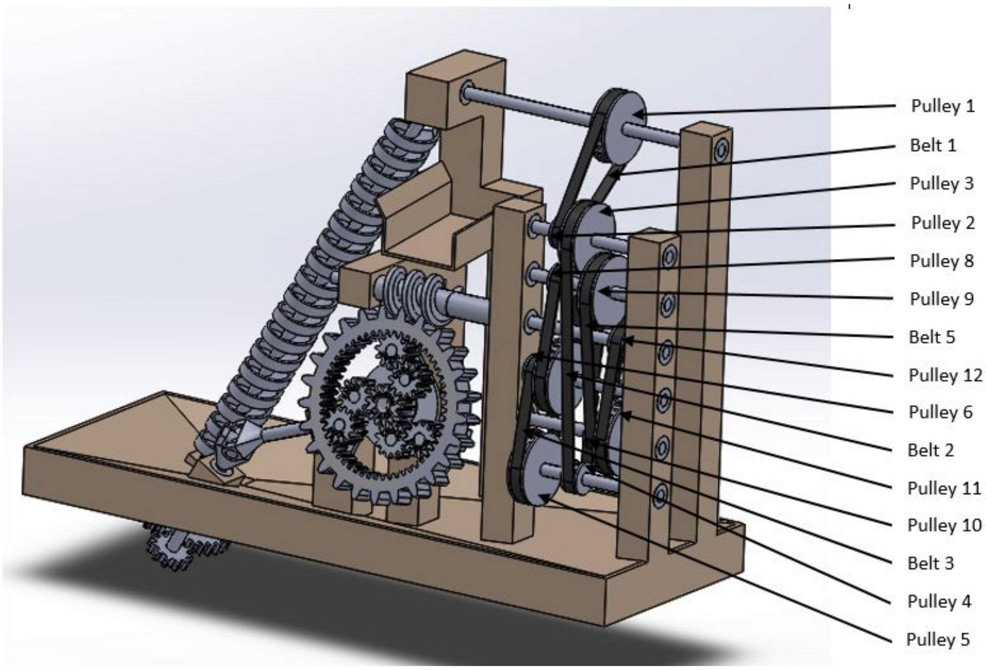

Figure 1: Belts and Pulleys Labeled

Below are Tables 2 and 3 which illustrate the calculations used for every belt to determine the lifespan.

Table 2: Beginning Calculations

Input Force(lb)	35.00	Pounds		Ratio 1	2.40	
Pulley 1 Diameter	2.40	Inches		Ratio 2	2.40	
Pulley 2 Diameter	1.00	Inches		Ratio 3	2.40	
Pulley 3 Diameter	2.40	Inches		Ratio 4	2.40	
Pulley 4 Diameter	1.00	Inches		Ratio 5	2.40	
Pulley 5 Diameter	2.40	Inches		Ratio 6	2.40	
Pulley 6 Diameter	1.00	Inches		Final Ratio	191.10	
Pulley 7 Diameter	2.40	Inches				
Pulley 8 Diameter	1.00	Inches		Shaft 1	0.26	Rpms
Pulley 9 Diameter	2.40	Inches		Shaft 2	0.63	Rpms
Pulley 10 Diameter	1.00	Inches		Shaft 3	1.51	Rpms
Pulley 11 Diameter	2.40	Inches		Shaft 4	3.62	Rpms
Pulley 12 Diameter	1.00	Inches		Shaft 5	8.68	Rpms
				Shaft 6	20.83	Rpms
				Final Output	50.00	Rpms

Table 3: Individual Belt Calculations

Belt 1							
Lifetime	10729.40						
	Number of Passes	70911.55					
	Belt Section - A						
		K	674.00				
		b	11.09				
	Tension 1	116.38					
		Force 1	24.71				
			Fc	0.00			
				Kc	0.56		
				V	0.16		
			Delta F	20.07			
				Hd	0.01		
				Nb	1.00		
				n	0.26		
				d	2.40		
			f sigma	1.67			
				C	6.04		
					Lp	17.90	
						L	16.60
						Lc	1.30
		Max Tensile Stress	91.67				
			Kb	220.00			
			Sheave Diameter	2.40			
	Tension 2	246.16					
		Force 1	26.16				
			Fc	0.00			
				Kc	0.56		
				V	0.16		
			Delta F	20.07			
				Hd	0.01		
				Nb	1.00		
				n	0.63		

				d	1.00		
			f sigma	1.46			
				C	8.15		
					Lp	17.90	
						L	16.60
						Lc	1.30
		Max Tensile Stress	220.00				
			Kb	220.00			
			Sheave Diameter	1.00			

References

R. G. Budynas, J. K. Nisbett, and J. E. Shigley, *Shigley's mechanical engineering design*, 10th ed. New York, NY: McGraw-Hill Education, 2015.